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Department of Mathematics, COMSATS Institute of Information Technology, Attock, Pakistan
Fellow of Bangladesh Academy of Science, University of Dhaka, Dhaka, Bangladesh
Institute of Future Environments, Queensland University of Technology, Gardens Point Campus, Queensland, Australia

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NATURAL CONVECTION FLOW WITH SURFACE RADIATION ALONG A VERTICAL WAVY SURFACE

S. Siddiqa¹, M. A. Hossain², and Suvash C. Saha³
1Department of Mathematics, COMSATS Institute of Information Technology, Attock, Pakistan
2Fellow of Bangladesh Academy of Science, University of Dhaka, Dhaka, Bangladesh
3Institute of Future Environments, Queensland University of Technology, Gardens Point Campus, Queensland, Australia

In this study, natural convection boundary layer flow of thermally radiating fluid along a heated vertical wavy surface is analyzed. Here, the radiative component of heat flux emulates the surface temperature. Governing equations are reduced to dimensionless form, subject to the appropriate transformation. Resulting dimensionless equations are transformed to a set of parabolic partial differential equations by using primitive variable formulation, which are then integrated numerically via iterative finite difference scheme. Emphasis has been given to low Prandtl number fluid. The numerical results obtained for the physical parameters, such as, surface radiation parameter, $\tilde{R}$, and radiative length parameter, $\xi$, are discussed in terms of local skin friction and Nusselt number coefficients. Comprehensive interpretation of velocity distribution is also given in the form of streamlines.

1. INTRODUCTION

Roughened surfaces are encountered in several heat transfer devices, such as flat plate solar collectors and flat plate condensers in refrigerators. Larger scale surface nonuniformities are encountered, for example, in cavity wall insulating systems, grain storage containers and industrial heat radiators. Initially, the effects of such nonuniformities on the vertical convective boundary layer flow of a Newtonian fluid was analyzed by Yao [1] and Moullic and Yao [2, 3]. Hossain and Pop [4] have investigated the magnetohydrodynamic boundary layer flow and heat transfer from a continuous moving wavy surface, and solved the nonsimilar set of boundary layer equations numerically with Keller box method. However, the problem of free convection flow from a wavy vertical surface in the presence of a transverse magnetic field was studied by Alam et al. [5]. Later, Hossain and Rees [6] investigated the combined effect of thermal and mass diffusion on the natural convection flow of a viscous incompressible fluid along a vertical wavy surface. Jang et al. [7] and Jang...
and Yan [8], respectively, investigated the effects of heat and mass transfer for the natural and mixed convection flow along a vertical wavy surface. In addition, numerical study of laminar forced convection in entrance region of a wavy wall channel was analyzed by Mohamed et al. [9]. They solved the governing equations numerically through iterative finite volume scheme, and discussed the results for various important physical parameters. Cheng et al. [10] performed well the 3-D numerical analysis of periodically developed fluid flow and heat transfer characteristics in the triangular wavy fin-and-tube heat exchanger based on the field synergy principle. Particularly, the influence of the wavy angle, fin pitch, tube diameter, and wavy density on pressure drop and heat transfer characteristics are provided in their article under different Reynolds numbers. The linear and nonlinear eddy viscosity models were examined by Assato and Lemos [11] in sinusoidal wavy channels to predict the turbulent flows. Numerical simulation of Al₂O₃/water nanofluids natural convection has been performed by Abu-Nada and Oztop [12] in a wavy walled cavity. They transformed the governing equations in streamfunction vorticity form and solve them numerically by adopting a finite volume technique. More recently, numerical study on water cooled single layer and double layer wavy microchannel heat sink was conducted by Xie et al. [13], and they discussed the results in terms of Nusselt number.

Heat transfer for sufficiently high temperature surfaces needs a simultaneous analysis of the influence of several kinds of heat transfer mechanisms. One such mechanism through which heat can be transferred more rapidly is by the absorption, emission, and scattering of radiation by the fluid. Radiation effects are important in the context of space technology and processes involving high temperatures. Ozisik [14], Sparrow and Cess [15], Cess [16], and Arpaci [17] initially studied the interaction of thermal radiation. Later, considering the Rosseland diffusion approximation, investigations on the natural convection flow as well as on the mixed convection flow

<table>
<thead>
<tr>
<th>NOMENCLATURE</th>
<th>x, y</th>
<th>dimensionless Cartesian coordinates</th>
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<tbody>
<tr>
<td>$C_f$</td>
<td>coefficient of local skin friction</td>
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<td>$g$</td>
<td>acceleration due to gravity, m/s²</td>
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<td>$Gr_L$</td>
<td>Grashof number</td>
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<tr>
<td>$L$</td>
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<td>$Pr$</td>
<td>Prandtl number</td>
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<tr>
<td>$R$</td>
<td>surface radiation parameter</td>
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<td>temperature at the surface, K</td>
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<td>temperature of the ambient fluid, K</td>
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<td>$u$, $v$</td>
<td>dimensional fluid velocities in the $x$- and $y$-directions, respectively, m/s</td>
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<td>$\zeta$</td>
<td>radiative length parameter</td>
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Subscripts

| $w$ | wall condition |
| $\infty$ | ambient condition |
of an optically dense gray viscous fluid past or along heated bodies of different geometries, such as, vertical and horizontal flat plate, cylinder, sphere, wavy surface and axisymmetric rotating and non-rotating bodies under different boundary conditions, have been accomplished by Hossain et al. [18, 19], Hossain and Munir [20], Hossain and Rees [21], Molla and Hossain [22], and Siddiqa et al. [23]. In these analyses consideration has been given to gray gases that emit and absorb, but do not scatter, thermal radiation.

On the other hand, heat transfer can also be determined for a nonabsorbing medium in which radiation absorption, emission, and scattering processes are neglected and the surface of the object uniformly emits into the surrounding medium a constant thermal flux, which is carried off as convective-conductive and radiative components. In this regard, Martynenko et al. [24] examined the influence of thermal radiation on the natural convective flow of a vertical surface situated in a nonabsorbing medium. Later, Salomatov and Puzyrev [25] investigated the influence of thermal radiation on the laminar natural and forced convection boundary layer flow of a nonabsorbing fluid with variable thermophysical properties flowing around a heat emitting surface and obtained the solutions for the small and large values of a thermal radiation parameter. Further, Sokovishin and Shapiro [26] analyzed the effect of thermal radiation on natural convective heat liberation from the surface of a vertical cylinder located in a transparent medium. They adopted a finite difference method in order to obtain the solution of the problem in terms of rate of heat transfer. Very recently, Siddiqa and Hossain [27] studied mixed convection boundary layer flow over a vertical flat plate in a nonabsorbing medium, and obtained the solutions numerically with the help of a finite difference method along with Gaussian elimination technique.

To the best of our knowledge, natural convection boundary layer flow of thermally radiating fluid along a heated vertical wavy surface has not been analyzed yet in a nonabsorbing medium. Therefore, the primary motivation of the work is to examine the above-mentioned problem. It is considered that the object is located in the optically transparent medium, and processes of radiation absorption, emission, and scattering are neglected. It is, however, noted that the relationship between convection and thermal radiation is developed with the help of a boundary condition of second kind on the thermally radiating vertical wavy surface. It needs to be mentioned here that in radio electronic devices, the measurement of the thermal regimes necessitate the inquiry of energy transfer from high temperature elements to the surrounding medium. Moreover, it is also necessary to examine not only the influence of transverse curvature on heat transfer but also the interaction of various forms of heat transfer while calculating heat liberation from the surfaces of bodies of semiconductor devices, thermo-resistors, micro-conductors, etc. Particularly, in the field of electronics the transfer of thermal energy to an immobile medium is in practice with the help of two modes; namely, natural convection and radiation. Therefore, the present study also aims to look at the simultaneous effects of thermal radiation and natural convection in a nonabsorbing medium. The governing boundary layer equations are reduced to parabolic partial differential equations due to the introduction of primitive variable formulation (PVF), which in principle can be integrated numerically by employing a straightforward finite difference method in contrast with the Gaussian elimination method. Numerical results are obtained with regard to
local skin friction coefficient and local Nusselt number coefficient for multifarious parameters that occur while addressing the issue. Particularly, the numerical results are presented for liquid metals which are deliberately served in industries as a coolant. In addition to it, streamlines are also graphed so that one can understand the flow pattern, followed by the fluid in the underlying situation.

2. MATHEMATICAL FORMULATION

Consider the steady two-dimensional natural convection flow of a viscous incompressible gas along a semi-infinite vertical wavy surface situated in the optically transparent medium. The processes of radiation absorption, emission, and scattering are neglected. Further assume that the surface temperature of the wavy surface is held constant at $T_w$, which is greater than ambient fluid temperature $T_1$. All the fluid properties are assumed constant except density which, according to the usual Boussinesq approximation, varies linearly with temperature. The boundary layer analysis outlined below allows $\tilde{y}_w = \tilde{\sigma}(\tilde{x})$ being arbitrary, but our detailed numerical work will assume that the surface exhibits sinusoidal deformations. Therefore, the shape of the wavy surface profile is assumed to pursue the following pattern:

$$\tilde{y}_w = \tilde{\sigma}(\tilde{x}) = \alpha \sin \left( \frac{2\pi x}{L} \right)$$

(1)

Where $\alpha$ is the dimensionless amplitude of wavy surface, $L$ the characteristic length scale associated with the waves, and $\omega$ the number of sinuosity.

The fundamental boundary layer equations for steady flow may now be written as follows:

$$\frac{\partial \tilde{u}}{\partial \tilde{x}} + \frac{\partial \tilde{v}}{\partial \tilde{y}} = 0$$

(2)

$$\tilde{u} \frac{\partial \tilde{u}}{\partial \tilde{x}} + \tilde{v} \frac{\partial \tilde{u}}{\partial \tilde{y}} = -\frac{1}{\rho} \frac{\partial \tilde{p}}{\partial \tilde{x}} + \nu \nabla^2 \tilde{u} + g \beta (\tilde{T} - T_\infty)$$

(3)

$$\tilde{u} \frac{\partial \tilde{v}}{\partial \tilde{x}} + \tilde{v} \frac{\partial \tilde{v}}{\partial \tilde{y}} = -\frac{1}{\rho} \frac{\partial \tilde{p}}{\partial \tilde{y}} + \nu \nabla^2 \tilde{v}$$

(4)

$$\tilde{u} \frac{\partial \tilde{T}}{\partial \tilde{x}} + \tilde{v} \frac{\partial \tilde{T}}{\partial \tilde{y}} = \alpha \frac{\partial^2 \tilde{T}}{\partial \tilde{y}^2}$$

(5)

Where $\tilde{u}, \tilde{v}$ are the velocity components in the $\tilde{x}$ and $\tilde{y}$ directions, respectively, in the momentum boundary layer, $\nu$ the kinematic coefficient of viscosity, $\rho$ the density of the fluid, $g$ identifies the gravitational force, $\beta$ the coefficient of thermal expansion, $\alpha$ the thermal diffusivity, $p(\sim)$ the pressure and $\tilde{T}$ the temperature of the fluid in the thermal boundary layer. The coordinate system and the flow configuration of the problem are shown in Figure 1.
The present work also consider that the wavy surface transmit uniform thermal flux, \( q_w \), into the surrounding medium. Moreover, the relationship between convection and thermal radiation is developed with the help of a boundary condition of second kind on the vertical surface which is supposed to be a gray diffusion radiator with emissivity \( \varepsilon \). The radiative heat flux component on the wall is expressed with the help of Stefan-Boltzmann law. Therefore, the boundary conditions can be written as follows (see [24]).

\[
\begin{align*}
\tilde{y} = \tilde{y}_w = \sigma(\tilde{x}) : & \quad \tilde{u} = \tilde{v} = 0, \quad \kappa (n \cdot \nabla \tilde{T}) = -q_w + \varepsilon \sigma_v (T_w^4 - T_\infty^4) \\
\tilde{y} \to \infty : & \quad \tilde{u} \to 0, \quad \tilde{T} \to T_\infty
\end{align*}
\]

Where \( \sigma_v \) is the Stefan-Boltzmann constant, \( \kappa \) the thermal conductivity of the fluid, and \( n \) the unit vector normal to the wavy surface.

The governing equations are transformed into dimensionless forms upon incorporating the following suitable variables:

\[
\begin{align*}
\chi = \frac{\tilde{x}}{L}, \quad y = \frac{\tilde{y} - \sigma(\tilde{x})}{L} \text{Gr}_L^{1/5}, \quad \tilde{u} = \frac{\nu}{L} \text{Gr}_L^{2/5} u, \quad (\tilde{y} - \sigma_v \tilde{u}) = \frac{\nu}{L} \text{Gr}_L^{1/5} v \\
\tilde{p} = \frac{\nu \rho^2}{L^2} \text{Gr}_L^{4/5} p, \quad \tilde{T} - T_\infty = \frac{q_w L}{\kappa \text{Gr}_L^{-1/5} 0}, \quad \sigma_x = \frac{d\tilde{\sigma}}{d\chi} = \frac{d\sigma}{dx}
\end{align*}
\]
which leads to the following dimensionless equations:

\[ \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \]  

(8)

\[ u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = -\frac{\partial p}{\partial x} + \sigma_x \text{Gr}^{1/5}_L \frac{\partial p}{\partial y} + (1 + \sigma_x^2) \frac{\partial^2 u}{\partial y^2} + 0 \]  

(9)

\[ \sigma_x \left( u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \right) + \sigma_{xx} u^2 = -\text{Gr}^{1/5}_L \frac{\partial p}{\partial y} + \sigma_x (1 + \sigma_x^2) \frac{\partial^2 u}{\partial y^2} \]  

(10)

\[ u \frac{\partial \theta}{\partial x} + v \frac{\partial \theta}{\partial y} = \frac{1}{Pr} (1 + \sigma_x^2) \frac{\partial^2 \theta}{\partial y^2} \]  

(11)

Boundary conditions take the following form:

\[ y = 0 : \quad u = v = 0, \quad (1 + \sigma_x^2)^{1/2} \left( \frac{\partial \theta}{\partial y} \right) = -1 + \xi(2 + R\xi0)(2 + 2R\xi0 + R^2\xi^20^2) \]

\[ y \to \infty : \quad u \to 0, \quad \theta \to 0 \]  

(12)

In Eqs. (9)–(12), Gr\(_L\) is the local Grashof number which measures the ratio of thermal diffusion to the viscous forces acting on the radiating fluid. R is the surface radiation parameter that measures the ratio of total heat flux transported from the surface of the wall to the radiative component. Also, \( \xi \) is termed as radiative length parameter that measures the degree of intensity of the ratio of radiative component and Grashof number. It should be noted that for \( \xi = 0 \), the surface becomes non-radiating. Thus, \( \xi \) establishes the connection between the radiative and convective components of the heat flux. Further, the dimensionless Prandtl number, Pr, calculates the strength of momentum diffusivity to the thermal diffusivity. Mathematical expressions for these quantities are given in Eq. (13).

\[ \text{Gr}_L = \frac{g\beta q_w L^4}{\kappa \nu^2}, \quad R = \frac{q_w}{\sigma e T^4}, \quad \xi = \frac{\sigma e T^3 L}{\kappa \text{Gr}_L^{1/5}}, \quad \text{Pr} = \frac{\nu}{\alpha} \]  

(13)

Equation (10) indicates that the pressure gradient along the y direction is \( O(\text{Gr}_L^{1/5}) \), which implies that the lowest order pressure gradient along the x direction can be determined from the inviscid-flow solution. In the present problem, this pressure gradient is zero because there is no externally induced free stream. Equation (10) further shows that \( \text{Gr}_L^{1/5} \partial p/\partial y \) is \( O(1) \), and is determined by the left-hand side of this equation. Thus, the elimination of \( \partial p/\partial y \) from Eqs. (9) and
leads to the following equation:

\[
\frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = (1 + \sigma_x^2) \frac{\partial^2 u}{\partial y^2} - \frac{\sigma_x \sigma_{xx}}{1 + \sigma_x^2} u^2 + \frac{1}{1 + \sigma_x^2} \theta
\]  

(14)

Now, primitive variable formulation (PVF) is introduced that switches the system of Eqs. (8), (11), (12), and (14) into a convenient form. To obtain the transformed equations, consider the following continuous formulations:

\[
x = X, \quad Y = \frac{y}{x^{1/5}}, \quad U = \frac{u}{x^{1/5}}, \quad V = x^{1/5}v, \quad \Theta = \frac{\theta}{x^{1/5}}
\]  

(15)

Introducing Eq. (15) into the Eqs. (8), (11), (12), and (14), the following equations are obtained:

\[
X \frac{\partial U}{\partial X} - Y \frac{\partial U}{\partial Y} + 3 U \frac{\partial V}{\partial Y} = 0
\]  

(16)

\[
UX \frac{\partial U}{\partial X} + \left( V - \frac{UY}{5} \right) \frac{\partial U}{\partial Y} + \left( \frac{3}{5} + \frac{X \sigma_x \sigma_{xx}}{(1 + \sigma_x^2)} \right) U^2 = \left( 1 + \sigma_x^2 \right) \frac{\partial^2 U}{\partial Y^2} + \frac{\Theta}{(1 + \sigma_x^2)}
\]  

(17)

\[
UX \frac{\partial \Theta}{\partial X} + \left( V - \frac{UY}{5} \right) \frac{\partial \Theta}{\partial Y} + \frac{U \Theta}{5} = \frac{1}{Pr} \left( 1 + \sigma_x^2 \right) \frac{\partial^2 \Theta}{\partial Y^2}
\]  

(18)

Transformed boundary conditions are as follows:

\[
Y = 0 : \quad U = V = 0 \quad (1 + \sigma_x^2)^{1/2} \left( \frac{\partial \Theta}{\partial Y} \right) = -1 + \xi \Theta X^{1/5} (2 + R \xi \Theta X^{1/5})
\]  

(19)

Now, Eqs. (16)–(18) subject to the boundary conditions (19) are discretized by a central-difference scheme for the diffusion terms and a backward-difference scheme for the convection terms. The resulting implicit tridiagonal algebraic system is solved by Thompson’s method.

The computation has been started at \( X = 0.0 \), and then marched up to \( X = 4.0 \) using the step size, \( \Delta X = 0.001 \). At every \( X \) station, the computations are iterated until the difference of the results, of two successive iterations become less or equal to \( 10^{-6} \). In order to get accurate results, we have compared the results at different grid size in \( Y \) direction and reached at the conclusion to chose \( \Delta Y = 0.05 \). In this integration, the maximum value of \( Y \) is taken to be 30.0. Its worth mentioning that the method has also been used by Siddiqa and Hossain [28] successfully and accurately.

The physical measurements like local skin friction coefficient, \( C_f \), and local Nusselt number coefficient \( NuGr^{-1/5} \) can be calculated at this moment due to the fact that \( U, \Theta \), and their derivatives are now known. These quantities are much significant from an engineering point of view, as both can be served to improve specifically the efficiency and shape of many equipments in aerodynamics. Below, are the expressions for these physical quantities, respectively.
\[
C_f \text{Gr}_L^{3/5} = X^{2/5} \frac{\partial U}{\partial Y} \\
\text{NuGr}_L^{-1/5} = X^{-1/5} \frac{1}{\Theta_u(1 + \sigma X)^{1/2}} \\
\left(1 - \xi \Theta X^{1/5}(2 + R\xi \Theta X^{1/5})(2 + 2 R\xi \Theta X^{1/5} + R^2\xi^2 \Theta^2 X^{2/5}) \right)
\]

One can infer from the relations given in Eq. (20) that the local skin friction coefficient and the local Nusselt number coefficient varies locally with the parameter, \(X\).

In the light of the previous investigations, we noted that for the nonradiating case, i.e., when \(\xi = 0\), the present problem reduces to the one investigated by Moulic and Yao [2], in which the full numerical solutions are obtained with the help of efficient numerical scheme. For the sake of comparison, some results from the work of Moulic and Yao [2] are shown in Figure 2, which are found to be excellent. This clearly indicates that the present numerical scheme is quite accurate.

3. RESULTS AND DISCUSSION

In this article, the effect of thermal radiation on natural convection flow along a vertical wavy surface is analyzed in the nonabsorbing medium. The wavy surface is located in the optically transparent medium and, particularly, the processes of
radiation absorption, emission, and scattering are neglected. The relationship between convection and thermal radiation is, however, established with the aid of second kind of boundary condition on the thermally radiating wavy surface. The interaction of various forms of heat transfer is much important while calculating heat liberation from the surfaces of bodies of semiconductor devices, thermoresistors, micro-conductors, etc. Keeping this application in mind, the present study is initiated which is not discussed yet in the literature for the nonabsorbing medium. The dimensionless boundary layer equations are transformed into suitable form for numerical integration. An efficient finite difference scheme is applied in connection with Gaussian elimination method. Numerical results are plotted for several physical parameters in order to know the behavior of local skin friction coefficient and local Nusselt number coefficient. In addition, streamlines are also shown to analyze the flow pattern.

Liquid metals can be used in a range of applications because they are non-flammable, non-toxic and environmentally safe. This is why liquid metals have number of technical applications in source exchangers, electronic pumps, ambient heat exchangers, and also used as heat engine fluids. Its technical importance and tempting applications in industries motivate us to consider liquid metals here as well. Therefore, Prandtl number Pr is taken to be 0.05, which is appropriate for lithium at typical operating temperatures.

It is worthy to mention that the set of Eqs. (16)–(19) with $\xi = 0$, reduces to the model investigated by Moulic and Yao [2]. They tackled the problem numerically by employing the finite difference method. The present numerical results are reduced for wall temperature function, $\theta_w$, to those discussed by Moulic and Yao [2] by taking $\xi = 0$, $R = 0$, $Pr = 1.0$, and $\alpha = 0.0$, 0.1, and 0.3. One can see an excellent agreement between the present results and those known from the literature [2].

The results are also compared in tabular form with Pop et al. [29] for Newtonian case only. In Table 1, numerical values of surface temperature, $\theta_w$, for $\alpha = 0.0$, 0.1 and 0.2, $Pr = 1.0$, $R = 0.0$, and $\xi = 0.0$ are entered for comparison. Here, also, the

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comparison shows excellent agreement between the present and the already published results for the particular values of $X$.

The influence of surface radiation parameter $R$ is discussed initially on local skin friction coefficient $C_f \text{Gr}_L^{-3/5}$ and local Nusselt number coefficient $\text{NuGr}_L^{-1/5}$, in Figure 3 for $R = 0.0$, $1.0$, $2.0$, $5.0$, and $10.0$, while other parameters are $Pr = 0.05$, $\xi = 0.2$, and $\alpha = 0.3$. It can be seen from these figures that the coefficient of local skin friction and the coefficient of local Nusselt number decreases considerably, owing to the increase in the surface radiation parameter $R$. The amplitude of the wave is also reduced as surface radiation parameter $R$ increases from 0 to 10, which indicates that $R$ lowers the magnitude of the frictional forces. In the case of the skin friction coefficient, the waves develop slowly and amplitude increases as the flow moves away from the leading edge; while on the other hand, for the

![Figure 3. (a) Variation of local skin friction coefficient, and (b) local Nusselt number coefficient for $R = 0.0$, $1.0$, $2.0$, $5.0$, and $10.0$ while $\xi = 0.2$, $\alpha = 0.3$, and $Pr = 0.05$ (color figure available online).](image-url)
coeficient of Nusselt number the wavy pattern gradually decays down. It is also observed that variation in $R$ does not effect the momentum and thermal boundary layer thicknesses.

Likewise, in Figure 4 the effect of radiative length parameter $\xi$ is shown graphically on the local skin friction coefficient $C_f\text{Gr}_L^{-3/5}$ and local Nusselt number coefficient $\text{NuGr}_L^{-1/5}$ against the axial distance $X$, which ranges from 0.0 to 4.0. In this figure, $\xi$ takes the values 0.0, 0.25, and 0.5 for $R = 2.0$, $\alpha = 0.3$, and $Pr = 0.05$. For $\xi = 0.0$, the results reduced to those valid for nonradiating surface. It is observed that both the coefficient of local skin friction and the coefficient of local Nusselt number decreases considerably owing to the increase in the value of $\xi$. The figure clearly indicates that the amplitude of the waves diminishes due to the enhancement in

![Figure 4](image-url)
the radiative length parameter; therefore, $\xi$ plays an important role in the establishment of the asymptotic behavior of the fluid flow. It is also observed that momentum and thermal boundary layer thicknesses reduce as $\xi$ gets stronger.
Streamlines are drawn in Figure 5 for various values of surface radiation parameter $R$ against the axial distance $x$. In this figure, $R$ takes the values 0.0, 3.0, and 6.0, whereas radiative length parameter $\xi$, amplitude of the wavy surface $\alpha$ and $Pr = 0.05$, $R = 2.0$, and $\alpha = 0.3$.

Figure 6. Streamlines plot for (a) $\xi = 0.0$, (b) $\xi = 0.2$, and (c) $\xi = 0.4$, while $Pr = 0.05$, $R = 2.0$, and $\alpha = 0.3$.

Streamlines are drawn in Figure 5 for various values of surface radiation parameter $R$ against the axial distance $x$. In this figure, $R$ takes the values 0.0, 3.0, and 6.0, whereas radiative length parameter $\xi$, amplitude of the wavy surface $\alpha$ and
Prandtl number Pr are set to be 0.2, 0.3, and 0.05, respectively. Here also, it is observed that due to the increment in surface radiation parameter, the velocity field decreases, which clearly suggests that $R$ lowers the magnitude of frictional forces. Therefore, $R$ acts as a resisting force which opposes the motion of the fluid.

Finally, in Figure 6 the results are shown for several values of radiative length parameter, $\xi$ in terms of streamlines. The radiative length parameter $\xi$ is chosen to be 0.0, 0.2, and 0.4, while other parameters are $R = 2.0$, $\alpha = 0.3$, and $Pr = 0.05$. The result produced for $\xi = 0.0$ corresponds to the model discussed by Moulic and Yao [2], while for non-zero $\xi$ the medium is considered to be nonabsorbing. It is worth mentioning that radiative length parameter also opposes the motion of the fluid. Therefore, from the present analysis it can be concluded that a nonabsorbing medium decelerates the fluid flow.

4. CONCLUSION

In this work, the effect of thermal radiation on natural convection flow along a semi-infinite wavy surface is analyzed in a nonabsorbing medium. This analysis actually considers the optically transparent medium and neglected the radiation absorption, emission, and scattering. The relationship between convection and thermal radiation is, however, established with the aid of a second kind of boundary condition on the thermally radiating vertical wavy surface. The governing boundary layer equations are reduced to parabolic partial differential equations, which in principle can be integrated numerically by employing a straightforward finite difference scheme in contrast with the Gaussian elimination method. The numerical results are acquired in terms of local skin friction coefficient and local Nusselt number coefficient for different values of the parameters that occur while modeling the problem. Particularly, the numerical results are presented for liquid metals, which are frequently used in industries as a coolant. In addition, streamlines are also graphed so that one can understand the flow pattern followed by the fluid in the underlying situation. From the above investigation, we may conclude that the coefficient of local skin friction and the coefficient of local Nusselt number diminishes owing to the increase in the surface radiation parameter $R$ and radiative length parameter $\xi$.

REFERENCES


