NATURAL CONVECTION IN A TRIANGULAR ENCLOSURE SUBJECT TO PERIODIC THERMAL FORCING

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Abstract

The effects of periodic thermal forcing on the flow field and heat transfer through an attic space are examined numerically in this paper. We consider the case with a fixed aspect ratio of 0.5 and a fixed Grashof number of $1.33 \times 10^6$. The numerical results reveal that, during the daytime, the flow is stratified; whereas at the night-time, the flow becomes unstable. A number of regular plumes and vortices are observed in the contours of isotherms and stream functions respectively. Moreover, the flow appears to be symmetric during the daytime, and becomes asymmetric at the night-time. It is also found that the flow is weaker during the daytime than that at the night-time in the present case, and the calculated heat transfer rate at the night-time is approximately three times greater than the heat transfer rate during the daytime.

Nomenclature

- $A$: aspect ratio, $H/L$
- $Gr$: Grashof number, $g\beta T_A H^3/\nu^2$
- $g$: acceleration due to gravity
- $H$: enclosure maximum height
- $L$: enclosure horizontal half-length
- $Pr$: Prandtl Number, $\nu/\kappa$
- $p$: pressure
- $P$: period of thermal forcing
- $t$: time
- $T$: temperature
- $T_A$: temperature amplitude
- $T_0$: operating temperature
- $Nu$: Nusselt number
- $q$: convective heat flux
- $k$: thermal conductivity
- $h_{eff}$: heat transfer coefficient
- $u, v$: velocity components along $x$- and $y$-axes.
- $x, y$: horizontal and vertical coordinates
- $\beta$: coefficient of thermal expansion
- $\rho$: density of the fluid
- $\nu$: kinematic viscosity
- $\kappa$: thermal diffusivity

1. Introduction

Heat transfer through an attic space into or out of buildings is an important issue for attic shaped houses in both hot and cold climates. One of the important objectives for design and construction of houses is to provide thermal comfort for occupants. In the present energy-conscious society, it is also a requirement for houses to be energy efficient, i.e. the energy consumption for heating or air-conditioning houses must be minimized. Relevant to these objectives, research into heat transfer in attics has been conducted for more than two decades. Initially, the focus of the research was to obtain previously unavailable heat transfer data for a triangular enclosure heated or cooled from below. Flack (1980) adopted an isosceles triangle for his experimental model, and conducted flow visualizations and heat transfer measurements for night-time (heating from below) conditions. The heat transfer data was obtained using a Wollaston prism schlieren interferometer. In addition, flow velocities at selected locations were measured using a laser velocimeter. The velocity measurements were made primarily to aid the general understanding of the flow structure. It was found that, at low Grashof numbers, the flow remained laminar. However, as the Grashof number was increased, the
flow eventually became turbulent. Flack (1980) reported that the transition from laminar to turbulent regimes took place at critical Grashof numbers $Gr_c = 3.01 \times 10^5$, $8.88 \times 10^5$, and $10.50 \times 10^5$ for aspect ratios $A = 0.58$, 1.0 and 1.73 respectively. Furthermore, Flack (1980) reported that four Bénard type convective cells were present in the laminar flow regime, but there was no mention of further details of the cellular flow pattern, or the relative positions of the cells.

The attic problem under the night-time conditions was again investigated experimentally by Poulikakos and Bejan (1983a). In their study, they modelled the enclosure as a right-angled triangle with an adiabatic vertical wall, which corresponded to half of the full attic domain. A fundamental study of the fluid dynamics inside an attic-shaped triangular enclosure subject to the night-time conditions was performed by Poulikakos and Bejan (1983b) with an assumption that the flow was symmetric about the centre plane. Salmun (1995a) considered the same problem inside a two-dimensional triangular geometry filled with air and water, respectively, with various aspect ratios and for Rayleigh numbers ranging between $10^2$ and $10^5$. The stability of the reported single-cell steady state solution (Poulikakos and Bejan 1983b) was re-examined by Salmun (1995b) who applied the same procedures developed by Farrow and Patterson (1993) for analysing the stability of a basic flow solution in a wedge-shaped geometry. Later Asan and Namli (2001) carried out an investigation to examine the details of the transition from a single cell to multi cellular structures. Haese and Teubner (2002) investigated the phenomenon for a large-scale triangular enclosure for night-time or winter day conditions with the effect of ventilation. The study of Holtzman et al. (2000) for the first time raised the issue of the symmetry of the flow about the mid-plane, and reported that at low Grashof numbers symmetric solutions are obtained, confirming the validity of the symmetry assumption in previous studies. However, at higher Grashof numbers, asymmetric solutions are obtained for different aspect ratios.

Unlike night-time conditions, the attic space problem under daytime (heating from above) conditions has received very limited attention. This may due to the fact that the flow structure in the attics subject to the daytime condition is relatively simple. The flow visualization experiments of Flack (1980) showed that the daytime flow remained stable and laminar for all the tested Grashof numbers (up to about $7 \times 10^6$). The first numerical study of the daytime attic problem was reported in Akinsete and Coleman (1982). For the purpose of air conditioning calculations, Asan and Namli (2000) have also reported numerical results for steady, laminar two-dimensional natural convection in a pitched roof of triangular cross-section under the summer day (daytime) boundary conditions.

It is seen from the above review that previous studies have considered either the constant heating or constant cooling boundary condition. In real situations however, buildings are subject to alternative heating and cooling over a diurnal cycle. Therefore, it is necessary to investigate the flow response and heat transfer in the attic space subject to a periodic thermal forcing, and this is the focus of this paper.

### 2. Model Formulation

Under consideration is a two-dimensional isosceles triangular enclosure (see Figure 1), which is $H = 0.14$ m high and $2L = 0.56$ m long, giving an aspect ratio of $A = H/L = 0.5$. The continuity, momentum and energy equations under the Boussinesq approximations to be solved are expressed as follows,

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0,$$

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = -\frac{1}{\rho} \frac{\partial p}{\partial x} + \nu \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right).$$
\[
\begin{align*}
\frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} &= -\frac{1}{\rho} \frac{\partial p}{\partial y} + v \left( \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right) + g\beta(T - T_0), \\
\frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} &= \kappa \left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right).
\end{align*}
\]

(3) (4)

Figure 1: A schematic of the geometry and boundary conditions of the enclosure

The boundary conditions for the present numerical simulation are also shown in Figure 1. Here, the temperature of the bottom wall of the cavity is fixed at \( T = T_0 = 295 \) K. A periodic temperature boundary condition is applied to the two inclined walls with a period of 2000 s and an amplitude of \( T_A = 5 \) K. The period is determined in consideration of the scaling predictions of Poulikakos and Bejan (1983b), which shows that the time for the adjustment of the temperature in the thermal boundary layer is by far shorter than the thermal forcing period of 24 hours in field situations. According to Poulikakos and Bejan (1983b), the growth time of the thermal boundary layer for the present model is of the order of 1s, and thus the selection of the period of 2000 s for the thermal forcing is deemed appropriate.

Initially, the fluid in the cavity is motionless and isothermal with a temperature of \( T_0 \). The interior fluid of the cavity is air. Based on these specified conditions, the Grashof number is calculated to be \( Gr = 1.33 \times 10^6 \) and the Prandtl number is \( Pr = 0.71 \) for the present simulation. This Grashof number is higher than the critical Grashof number Flack (1980) reported based on his experiments under the night-time condition. However, Flack (1980)'s experiments were conducted with constant temperatures on all boundaries, and in the present case, the temperatures on the sloping boundary are changing with time. Therefore, the critical Grashof numbers reported in Flack (1980) are not relevant to the present case. It is more likely that the flow would remain laminar at the present Grashof number during the diurnal cycle.

In order to avoid singularities at the tips in the numerical simulation, the tips are cut off by 5%, and an extra rigid non-slip and adiabatic vertical wall boundary is assumed near each tip. It is anticipated that this modification of the geometry will not alter the overall flow development significantly.

Equations (1) - (4) are solved along with the initial and boundary conditions using the SIMPLE scheme, in which the spatial derivatives are discretized with a second-order upwind scheme and the diffusion terms with a second order center-differenced scheme. The temporal derivatives are discretized with a second order implicit scheme. Mesh and time step dependence tests have been
Table 1: Parameters and results of mesh and time-step dependence test at $t = 0.3P$

<table>
<thead>
<tr>
<th>Mesh Size</th>
<th>Time Step(s)</th>
<th>Temperature at the three points in the cavity</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>(-0.10,0.03)</td>
</tr>
<tr>
<td>240×60</td>
<td>1.5</td>
<td>297.9595</td>
</tr>
<tr>
<td>360×80</td>
<td>1.0</td>
<td>297.9599</td>
</tr>
<tr>
<td>480×120</td>
<td>0.75</td>
<td>297.9605</td>
</tr>
<tr>
<td>720×160</td>
<td>0.5</td>
<td>297.9599</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Maximum variation (%)</td>
</tr>
</tbody>
</table>

carried out with four different meshes of 240×60, 360×80, 480×120 and 720×160 respectively. The time steps for these four meshes have been chosen in such a way that the CFL (Courant-Friedrich-Lewy) number remains the same for all four calculations. The results of the mesh and time-step dependence tests are shown in Tables 1 and 2, which show the temperatures recorded at three different positions in the cavity at specific times $t = 0.3P$ (600s) and $t = 0.75P$ (1500s) respectively. It is seen in Table 1 that the maximum variation of the calculated temperature among the four meshes is very small at $t = 0.3P$. Based on these tests, any of the tested meshes would be sufficient for resolving the flow. However, the calculated results at a later time of $t = 0.75P$ shows a maximum variation of about 4%. This is because at this time the flow is very unstable, and is dominated by rising and sinking plumes. Since the purpose of the present study is to describe the general flow response to the diurnal thermal forcing, rather than resolving the details of the flow instabilities, the finest mesh (720×160) and a time step of 0.5s are adopted.

Table 2: Parameters and results of mesh and time-step dependence test at $t = 0.75P$

<table>
<thead>
<tr>
<th>Mesh Size</th>
<th>Time Step(s)</th>
<th>Temperature at the three points in the cavity</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>(-0.10,0.03)</td>
</tr>
<tr>
<td>240×60</td>
<td>1.5</td>
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<tr>
<td>480×120</td>
<td>0.75</td>
<td>292.2433</td>
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<tr>
<td>720×160</td>
<td>0.5</td>
<td>292.4458</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Maximum variation (%)</td>
</tr>
</tbody>
</table>

3. Results and Discussion

In this section, the flow response to the periodic thermal forcing and the heat transfer through the bottom boundary are discussed for the case with $A = 0.5$, $Pr = 0.71$ and $Gr = 1.33\times10^6$.

3.1 Flow response to the periodic thermal forcing

Since the initial flow is assumed to be isothermal and motionless, there is a start-up component of the flow response. In order to minimise the start-up effect, three full thermal forcing cycles are calculated in the numerical simulation before consideration of the flow. It is found that the start-up effect for the present case is almost negligible, and the flow response in the third cycle is identical to that in the second cycle. In the following discussion, the results of the third cycle are presented. Figure 2 shows snapshots of streamlines and the corresponding temperature contours at different stages of the cycle. The plus (+) and minus (-) signs in the streamlines indicate anticlockwise and clockwise circulations of the flow respectively.

The flow and temperature structures shown in figure 2(a) for $t = 2.00P$ represent those at the beginning of the daytime heating process in the third thermal forcing cycle. At this time, the inclined surfaces and the bottom surface of the enclosure have the same temperature, but the
temperature inside the enclosure is lower than the temperature on the boundaries. It is seen that the residual temperature structure, which is formed in the previous cooling phase, is still present in figure 2 (a). The streamline contours in figure 2(a) show two circulating cells, and the temperature contours indicate stratification in the upper section of the enclosure.

As the upper surface temperature increases further, a distinct temperature stratification is established throughout the enclosure by the time $t = 2.05P$ (see figure 2b). The streamlines at this stage indicate that the centres of the two circulating cells have shifted closer to the inclined surfaces, indicating a strong conduction effect near those boundaries. This phenomenon has been reported previously in Akinsete and Coleman (1982) and Asan and Namli (2000) for the daytime condition with constant heating at the upper surface or constant cooling at the bottom surface.

At $t = 2.25P$, the temperature on the inclined surfaces peaks. Subsequently, the temperature drops, representing a decreasing heating effect. Since the interior flow is stably stratified prior to $t = 2.25P$, the decrease of the temperature at the inclined surface results in a cooling event, appearing first at the top corner and expanding downwards as the surface temperature drops further. At $t = 2.45P$, two additional circulating cells have formed in the upper region of the enclosure (see figure 2c), and the newly formed cells push the existing cells downwards. The temperature contours in figure 2(c) show two distinct regions, an expanding upper region responding to the cooling effect, and a shrinking lower region with stratification responding to the decreasing heating effect. By the time $t = 2.50P$ (figure 2d), the daytime heating ceases; the lower stratified flow region has disappeared completely; and the flow in the enclosure is dominated by the cooling effect. At this time, the top and the bottom surfaces again have the same temperature, but the interior temperature is higher than that on the boundaries.

As the upper inclined surface temperature drops below the bottom surface temperature ($t = 2.70P$, figure 2e), the cold-air layer under the inclined surfaces becomes unstable. At the same time, the hot-air layer above the bottom surface also becomes unstable. As a consequence, sinking cold-air plumes and rising hot-air plumes are visible in the isotherm contours and a cellular flow pattern is formed in the stream function contours (see figure 2e). It is also noticeable that the flow is asymmetric about the geometric symmetry plane. The large cell from the right hand side of the centreline is pushing the left cell towards the left tip and becoming larger. At the same time this large cell also changes its position. At $t = 2.95P$ (figure 2f) the large cell has crossed the centreline of the cavity and a small cell next to it moves into its position and grows.

In figure 2(g) two cells of approximately the same size stand beside the centreline of the cavity. The flow is then returning to symmetry and the number of cells is reducing quickly. There are only two cells remaining at the time $t = 3.00P$ (see figure 2h). Again the similar temperature and flow structures, like the beginning of the forcing cycle, are formed and the flow development is continued in the following cycle.

The horizontal velocity profiles and the corresponding temperature profiles evaluated along the line $x = L/2$ at different time instances of the third thermal forcing cycle are depicted in figure 3. At the beginning of the cycle the velocity is the highest near the roof of the attic (see figure 3a), which is the surface driving the flow. At the same time, the body of fluid residing outside the top wall layer moves fast toward the left. As time progresses, the velocity reduces. A three layer structure in the velocity field is found at $t = 2.45P$. After that time the flow completely reverses at $t = 2.50P$. 
Figure 2: Series of snapshots of stream function and temperature contours of the third cycle at different times. Left: streamlines; right: isotherms.
The corresponding temperature at the beginning of the cycle \( (t = 2.00P) \) is the same at both surfaces as shown in figure 3(b). However the temperature near the mid point of the line is lower than that at the surfaces by approximately 0.5 K, which is consistent with the previous discussion of the flow field. Subsequently the temperature of the top surface increases \( (t = 2.05P) \) while the bottom surface temperature is fixed. It is noteworthy that the top surface reaches its peak temperature at \( t = 2.25P \). By the time \( t =2.40P \), the top surface temperature has started to decrease. By comparing the temperature profiles at \( t = 2.05P \) and \( t = 2.45P \) shown in figure 3(b), it is clear that the temperatures at both the top and bottom surfaces are the same for these two time instances. However, different temperature structures are seen in the interior region. The same phenomenon has been found at the times \( t = 2.50P \) and \( t = 2.00P \).

In figure 3(c), the velocity profiles at the same location for the night-time condition are displayed. At \( t = 2.55P \) the velocity near the bottom surface is slightly higher than that near the top. Again a three layer structure of the velocity field is found at \( t = 2.65P \). However, this structure lasts only for a short time, and the same velocity structure as that at \( t = 2.55P \) appears at \( t = 2.75P \) and 2.85P. At the time of \( t = 3.00P \) the flow is totally reversed. The corresponding temperature profiles for the night-time condition are shown in figure 3(d). It is seen that the temperature lines are not as smooth...
as those observed for the daytime condition. At $t = 2.55P$, the temperature near the bottom surface decreases first and then increases with height. Near the top surface, the temperature decreases again rapidly. The behaviour near the bottom surface is because of the presence of a rising plume. Similar behaviour has been seen for $t = 2.65P$, $2.75P$ and $2.85P$. At $t = 3.00P$ again the bottom and top surface temperatures are the same with a lower temperature in the interior region.

3.2 Heat Transfer

The Nusselt number, which has practical significance, is calculated as follows:

$$\text{Nu} = \frac{h_{\text{eff}} H}{k} \quad (6)$$

where the heat transfer coefficient $h_{\text{eff}}$ is defined by

$$h_{\text{eff}} = \frac{q}{T_A} \quad (7)$$

Since the bottom surface temperature is fixed at $295K$ and the top surface temperature varies between $290K$ and $300K$, a zero temperature difference between the surfaces occurs twice in a cycle. Therefore, the amplitude of the temperature fluctuation ($T_A$) is chosen for calculating the heat transfer coefficient instead of a changing temperature difference, which would give an undefined value of the heat transfer coefficient at particular stages.

Figure 4 shows the thermal forcing (Figure 4(a)) and the calculated Nusselt number on the inclined surface of the cavity (Figure 4(b)). The time histories of the calculated Nusselt number exhibit certain significant features. Firstly, it shows a periodic behaviour in response to the periodic thermal forcing. Secondly within each cycle of the flow response, there is a time period with weak heat transfer and a period with intensive heat transfer. The weak heat transfer corresponds to the daytime condition and the intensive heat transfer corresponds to the night-time condition. At the night-time the flow is strongly dominated by convection. The calculated maximum Nusselt number is 5.95 at the night-time.

4. Conclusion

This paper has described the natural convection in an attic space with periodic thermal forcing. Some important features have been revealed from the present numerical simulation. It is found that the flow response to the temperature variation on the external surface is fast, and thus the start-up effect is almost negligible. The occurrence of sinking cold-air plumes and rising hot-air plumes in the isotherm contours and the formation of cellular flow patterns in the stream function contours confirm the presence of the Bénard-type instability. It is also observed that the flow undergoes a transition from symmetry to asymmetry and back to symmetry again about the geometric symmetry.
plane over a diurnal cycle for the Grashof number considered in this study. A three-layer velocity structure has been found along the line at $x = L/2$ in both the daytime heating and night-time cooling phases. Furthermore, the daytime responding flow is weak, whereas the night-time responding flow, which is dominated by convection, is intensive. The maximum value of the Nusselt number evaluated at the night-time is 5.95, whereas the maximum value of the Nusselt number in the daytime is 2.15.

References


