MHD natural convection flow from an isothermal horizontal circular cylinder under consideration of temperature dependent viscosity

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Abstract

Purpose – The purpose of this paper is to discuss, with numerical simulations, magnetohydrodynamic (MHD) natural convection laminar flow from an isothermal horizontal circular cylinder immersed in a fluid with viscosity proportional to a linear function of temperature.

Design/methodology/approach – The governing boundary layer equations are transformed into a non-dimensional form and the resulting nonlinear system of partial differential equations are reduced to convenient form, which are solved numerically by two very efficient methods: implicit finite difference method together with Keller box scheme; and direct numerical scheme.

Findings – Numerical results are presented by velocity and temperature distributions of the fluid as well as heat transfer characteristics, namely the shearing stress and the local heat transfer rate in terms of the local skin-friction coefficient and the local Nusselt number for a wide range of MHD parameter, viscosity-variation parameter and viscous dissipation parameter.

Originality/value – MHD flow in this geometry with temperature dependent viscosity is absent in the literature. In this paper, the results obtained from the numerical simulations have been verified by two methodologies.

Keywords Laminar flow, Convection, Viscosity, Natural convection, Magnetohydrodynamic, Temperature dependent viscosity, Horizontal circular cylinder

Paper type Research paper

Nomenclature

\( a \) = radius of the circular cylinder
\( C_p \) = specific heat at constant pressure
\( C_f \) = skin-friction coefficient
\( f \) = dimensionless stream function
\( g \) = acceleration due to gravity
\( Gr \) = Grashof number
\( k \) = thermal conductivity of the fluid
\( M \) = MHD parameter
\( Nu \) = Nusselt number
\( Pr \) = Prandtl number
1. Introduction

A study of the flow of electrically conducting fluid in presence of magnetic field is important from the technical point of view and such types of problems have received much attention by many researchers. Vajravelu and Hadjinicolaou (1997) studied the convective heat transfer in an electrically conducting fluid at a stretching surface. As mentioned by Vajravelu and Hadjinicolaou (1997), the rate of cooling and, therefore, the desired properties of the end product can be controlled by the use of electrically conducting fluids and the application of magnetic field. The use of magnetic field has been also used in the process of purification of molten metals from non-metallic inclusions. MHD free convection flow of visco-elastic fluid pasting an infinite porous plate was investigated by Chowdhury and Islam (2000). Raptis and Kafousian (1982) have investigated the problem of magnetohydrodynamic free convection flow and mass transfer through a porous medium bounded by an infinite vertical porous plate with constant heat flux. Moreover, Hossain and Ahmed (1990) and Hossain et al. (1997) discussed the both forced and free convection boundary layer flow of an electrically conducting fluid in presence of magnetic field. Combined effect of magnetic field and viscous dissipation on forced convection from flat plate and horizontal cylinder embedded in porous medium has been studied by El-Amin (2003a, b). Molla et al. (2005b, 2006b) have investigated the MHD natural convection flow on a sphere with uniform surface temperature and heat flux in presence of temperature dependent heat generation.

Natural convection flow of viscous incompressible fluid from a horizontal circular cylinder represents an important problem, which is related to numerous engineering applications such as to handle hot wire, steam pipe, etc. Sparrow and Lee (1976), looked at the problem of vertical stream over a heated horizontal circular cylinder. They obtain a solution by expanding velocity and temperature profiles in powers of $x$, the co-ordinate measuring distance from the lowest point on the cylinder. The exact solution is still out of reach due to the non-linearity in the Navier-Stokes equations. It appears that Merkin (1976, 1977), was the first who presented a complete solution of this problem using Blasius and Gortler series expansion method along with an integral method and a finite-difference scheme. Also the problem of free convection boundary layer flow on
cylinder of elliptic cross-section was studied by Merkin (1977). Ingham (1978), investigated the boundary layer flow on an isothermal horizontal cylinder. Hossain and Alim (1997) have investigated natural convection-radiation interaction on boundary layer flow along a vertical thin cylinder. Hossain et al. (1999), have studied radiation-conduction interaction on mixed convection from a horizontal circular cylinder. Recently, Nazar et al. (2002), have considered the problem of natural convection flow from lower stagnation point to upper stagnation point of a horizontal circular cylinder immersed in a micropolar fluid. Natural convection from a horizontal circular cylinder with heat generation has been investigated by Molla et al. (2006a).

All the above studies were confined to the fluid with constant viscosity. However, it is known that this physical property may change significantly with temperature. To predict accurately the flow behavior, it is necessary to take into account this variation of viscosity. On assuming that the viscosity of the fluid is linear functions of temperature, a semi-empirical formula was proposed by Charraudeau (1975) which is appropriate for small Prandtl number. Following him Hossain et al. (2000, 2002) investigated the natural convection flow past a permeable wedge, wavy surface and a flat plate for the fluid having temperature dependent viscosity and thermal conductivity. Molla and Hossain (2006) and Molla et al. (2009, 2005a) have investigated the natural convection flow from an isothermal horizontal circular cylinder and sphere with viscosity is the inverse linear functions of temperature, which is appropriate for the fluids having large Prandtl number.

In the present study, it is proposed to investigate the MHD natural convection flow of a viscous incompressible fluid having viscosity $\mu(T)$, which is the linear function of temperature, from an isothermal horizontal circular cylinder. The surface temperature $T_w$ of the cylinder is higher than that of the ambient fluid temperature $T_\infty$. In formulating the equations governing the flow the viscosity of the fluid has been assumed to be proportional to a linear function of temperature, a semi-empirical formula for the viscosity $\mu(T)$, as Charraudeau (1975). The governing partial differential equations are reduced to local non-similar partial differential forms by adopting appropriate transformations. The transformed boundary layer equations are solved numerically using very efficient finite-difference method (IFDM) known as Keller box technique (Keller, 1978; Cebeci and Bradshaw, 1984) and by direct numerical scheme (DNS). Effect of MHD parameter $M$ and viscosity-variation parameter $\gamma$, on the velocity and temperature distribution of the fluid as well as on the local rate of heat transfer in terms of the Nusselt number and the surface shearing stress in terms of the local skin-friction coefficient are shown graphically for fluid having Prandtl number $Pr = 0.73$.

2. Formulation of problem

A steady two-dimensional MHD laminar free convective flow from a uniformly heated horizontal circular cylinder of radius $a$, which is immersed in a viscous and incompressible fluid having temperature dependent viscosity. Here, viscosity is the linear function of the fluid temperature. It is assumed that the surface temperature of the cylinder is $T_w$, where $T_w > T_\infty$. Here, $T_\infty$ is the ambient temperature of the fluid, the configuration considered is as shown in Figure 1.

The equations governing the flow are:

$$\frac{\partial \hat{u}}{\partial \hat{x}} + \frac{\partial \hat{v}}{\partial \hat{y}} = 0 \tag{1}$$
The boundary conditions of equations (1)-(3) are:

\[ u = v = 0, \quad T = T_w, \text{ at } \hat{y} = 0 \] (4a)

\[ \hat{u} \to 0, \quad T \to T_\infty \text{ as } \hat{y} \to \infty \] (4b)

where \((\hat{u}, \hat{v})\) are velocity components along the \((\hat{x}, \hat{y})\) axes, \(g\) is the acceleration due to gravity, \(\rho\) is the density, \(\mu(T)\) is the viscosity of the fluid depending on the fluid temperature \(T\), \(\beta\) is the coefficient of thermal expansion, \(k\) is the thermal conductivity of the fluid, \(\alpha_0\) is the electrical conduction, \(\beta_0\) is the strength of magnetic field.

Out of the many forms of viscosity variation, which are available in the literature, we will consider only following form proposed by Charraudeau (1975):

\[ \mu = \mu_\infty [1 + \gamma^*(T - T_\infty)] \] (5a)

where \(\mu_\infty\) is the viscosity of the ambient fluid and \(\gamma^*\) is defined as follows:

\[ \gamma^* = \frac{1}{\mu_f} \left( \frac{\partial \mu}{\partial T} \right)_f \] (5b)

here \(f\) denotes the film temperature of the fluid.

We now introduce the following non-dimensional transformations:

\[ x = \frac{\hat{x}}{a}, \quad y = Gr^{1/4} \left( \frac{\hat{y}}{a} \right), \quad u = \frac{\rho a}{\mu_\infty} Gr^{-1/2} \hat{u}, \quad v = \frac{\rho a}{\mu_\infty} Gr^{-1/2} \hat{v}, \] (6)

\[ \theta = \frac{T - T_\infty}{T_w - T_\infty}, \quad Gr = \frac{g \beta (T_w - T_\infty) a^3}{\nu_\infty^2} \]

where \(\nu_\infty (= \mu_\infty/\rho)\) is the reference kinematic viscosity and \(Gr\) is the Grashof number and \(\theta\) is the non-dimensional temperature.

Substituting variables (6) into equations (1)-(3) leads to the following non-dimensional equations:

![Physical model and coordinate system](https://via.placeholder.com/150)
\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (7)
\]

MHD natural convection flow

\[
\frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = (1 + \gamma \theta) \frac{\partial^2 u}{\partial y^2} + \gamma \frac{\partial u}{\partial y} \frac{\partial \theta}{\partial y} + \theta \sin x - Mu \quad (8)
\]

With the boundary conditions (4) become:

\[
u = v = 0, \quad \theta = 1 \quad \text{at} \quad x = 0, \quad \text{for any} \quad y \quad (10a)
\]

\[
u = v = 0, \quad \theta = 1, \quad \text{at} \quad y = 0, \quad x > 0 \quad (10b)
\]

\[
u \to 0, \quad \theta \to 0, \quad \text{as} \quad y \to \infty, \quad x > 0 \quad (10c)
\]

where the viscosity-variation parameter \( \gamma \), magnetic parameter, Prandtl number Pr and viscous dissipation parameter are defined as, respectively:

\[
\gamma = \frac{1}{\mu_f} \left( \frac{\partial \mu}{\partial T_f} \right) (T_w - T_\infty), \quad M = \frac{\alpha_0 \beta_0 M_a^2}{\mu_\infty Gr^{1/2}}, \quad \text{Pr} = \frac{\mu_\infty c_p}{k} \quad \text{and} \quad Ec^* = \frac{g \beta a}{C_p} \quad (11)
\]

3. Solution methodology

Investigating the present problem we have employed two numerical methods, namely, implicit finite difference method (IFDM) with the Keller box scheme and the DNS, which are individually presented below.

3.1 Implicit finite difference method

IFDM, which was first introduced by Keller (1978) and elaborately describe by Cebeci and Bradshaw (1984). To solve equations (7)-(9), subject to the boundary conditions (10), we assume the following variables:

\[
\psi = xf(x,y), \quad \theta = \theta(x,y) \quad (12)
\]

where \( \psi \) is the non-dimensional stream function defined in the usual way as:

\[
u = \frac{\partial \psi}{\partial y}, \quad v = -\frac{\partial \psi}{\partial x} \quad (13)
\]

Substituting equations (12) and (13) into equations (8) and (9) we get, after some algebra the following transformed equations:

\[
\begin{align*}
(1 + \gamma \theta) & \frac{\partial^3 f}{\partial y^3} + f \frac{\partial^2 f}{\partial y^2} + \gamma \frac{\partial^2 f}{\partial y^2} \frac{\partial \theta}{\partial y} - \left( \frac{\partial f}{\partial y} \right)^2 + \frac{\theta \sin x}{x} - M \frac{\partial f}{\partial y} \\
= & x \left( \frac{\partial f}{\partial y} \frac{\partial^2 f}{\partial x \partial y} - \frac{\partial f}{\partial x} \frac{\partial^2 f}{\partial y^2} \right) \quad (14)
\end{align*}
\]
\[
\frac{1}{\text{Pr}} \frac{\partial^2 \theta}{\partial y^2} + \text{f} \frac{\partial \theta}{\partial y} + \text{Ec}^* X^2 (1 + \gamma \theta) \left( \frac{\partial \text{f}}{\partial y} \right)^2 = x \left( \frac{\partial \text{f}}{\partial y} \frac{\partial \theta}{\partial x} - \frac{\partial \theta}{\partial y} \frac{\partial \text{f}}{\partial x} \right)
\]  
(15)

Along with the boundary conditions:

\[
f = \frac{\partial \text{f}}{\partial y} = 0, \quad \theta = 1 \quad \text{at} \quad x = 0 \quad \text{any} \quad y
\]  
(16a)

\[
f = \frac{\partial \text{f}}{\partial y} = 0, \quad \theta = 1 \quad \text{at} \quad y = 0, \quad x > 0
\]  
(16b)

\[
\frac{\partial \text{f}}{\partial y} \to 0, \quad \theta \to 0, \quad \text{as} \quad y \to \infty, \quad x > 0
\]  
(16c)

The physical qualities of principle interest are shearing stress in terms of the skin-friction coefficient \( C_f \) and the rate of heat transfer in terms of the Nusselt number \( Nu \), which can be written in non-dimensional form as:

\[
C_f = \frac{\tau_w}{\rho U_\infty^2}, \quad Nu = \frac{a q_w}{k(T_w - T_\infty)}
\]  
(17)

where:

\[
\tau_w = \left( \mu \frac{\partial \text{u}}{\partial y} \right)_{\text{y}=0}, \quad q_w = -\text{k} \left( \frac{\partial T}{\partial y} \right)_{\text{y}=0}
\]  
(18)

Using the variables (6) and (12) and the boundary condition (16b), \( C_f \) and \( Nu \) take following forms:

\[
\frac{C_f}{Gr^{1/4}} = \frac{x}{1 + \gamma} \frac{\partial \text{f}(x,0)}{\partial y^2}
\]  
(19)

\[
\frac{Nu}{Gr^{-1/4}} = -\frac{\partial \theta(x,0)}{\partial y}
\]  
(20)

### 3.2 Direct numerical scheme

Here we introduce new transformations for the DNS method:

\[
X = x, \quad Y = y, \quad U = \frac{u}{x}, \quad V = v
\]  
(21)

Using equation (21) into equations (7) and (10), we get:

\[
X \frac{\partial U}{\partial X} + U + \frac{\partial V}{\partial Y} = 0
\]  
(22)

\[
XU \frac{\partial U}{\partial X} + V \frac{\partial U}{\partial Y} + U^2 = (1 + \gamma \theta) \frac{\partial^2 U}{\partial Y^2} + \gamma \frac{\partial U}{\partial Y} \frac{\partial \theta}{\partial Y} + \theta \sin X \frac{X}{X} - MU
\]  
(23)

\[
XU \frac{\partial \theta}{\partial X} + V \frac{\partial \theta}{\partial Y} = \frac{1}{\text{Pr}} \frac{\partial^2 \theta}{\partial Y^2} + \text{Ec}^* X^2 (1 + \gamma \theta) \left( \frac{\partial U}{\partial Y} \right)^2
\]  
(24)
The corresponding boundary conditions are:

\[
U = V = 0, \quad \theta = 1 \quad \text{at} \quad X = 0 \quad \text{any} \quad Y \quad \quad \quad (25a)
\]

\[
U = V = 0, \quad \theta = 1 \quad \text{at} \quad Y = 0, \quad X > 0 \quad \quad \quad (25b)
\]

\[
U \to 0, \quad \theta \to 0 \quad \text{as} \quad Y \to \infty, \quad X > 0 \quad \quad \quad (25c)
\]

Now equations (22)-(24) subject to the boundary conditions (25) are discretised for DNS using central-difference for diffusion terms and the forward-difference for the convection terms, finally we get a system of tri-diagonal algebraic equations which are solved by Gaussian elimination technique. The computation is started at \( X = 0.0 \), and then marches up to the upper stagnation point \( (X \approx \pi) \). Here \( \Delta x = \Delta y = 0.01 \) are used for the \( X \)- and \( Y \)-grids, respectively. We are at the position to measure of the physical quantities, namely the rate of heat transfer and the average rate of heat transfer which are important from the application point of view, from the following dimensionless relations:

\[
\frac{C_f Gr^{1/4}}{(1 + \gamma)} = X \left( \frac{\partial U}{\partial Y} \right)_{Y=0} \quad (26)
\]

\[
NuGr^{-1/4} = - \left( \frac{\partial \theta}{\partial Y} \right)_{Y=0} \quad (27)
\]

4. Results and discussion

In this paper, it has been investigated the problem of MHD laminar natural convection flow and heat transfer from an isothermal horizontal circular cylinder with temperature dependent viscosity where the viscosity of the fluid is proportional to the linear function of temperature, that means, if the temperature of the fluid increases, the viscosity of the fluid increases. This phenomenon occurs for small Prandtl number \( Pr \). For example, the viscosity of air is \( 0.6924 \times 10^{-5} \text{ kg m}^{-1} \text{s}^{-1} \), \( 1.3289 \text{ kg m}^{-1} \text{s}^{-1} \), \( 2.286 \text{ kg m}^{-1} \text{s}^{-1} \) and \( 3.625 \text{ kg m}^{-1} \text{s}^{-1} \) at 100, 200, 400 and 800 K temperature, respectively. The viscosity of ammonia \( \text{NH}_3 \) is \( 7.255 \times 10^{-6} \text{ kg m}^{-1} \text{s}^{-1} \), \( 12.886 \text{ kg m}^{-1} \text{s}^{-1} \) and \( 16.49 \text{ kg m}^{-1} \text{s}^{-1} \) at 220, 373 and 473 K, respectively (Cebeci and Bradshaw, 1984).

Equations (14) and (15) subject to the boundary conditions (16) are solved numerically using implicit finite IFDM together with Keller box and the equations (22)-(25) are solved by DNS. The numerical solutions start at the lower stagnation point of the cylinder, \( x = 0 \) and proceed round the cylinder up to the upper stagnation point, \( x \approx \pi \). Solutions are obtained for viscous dissipation parameter \( Ec^* = (0.0, 0.1, 0.2) \), MHD parameter \( M = (0.0, 0.2, 0.5, 0.8, 1.0) \) and for a wide range of values of the variable viscosity parameter \( \gamma = (0.0, 1.0, 3.0, 5.0) \). Since the values of \( f''(x, 0) \) or \( (\partial U/\partial Y)_{Y=0} \) and \( \theta'(x, 0) \) or \( (\partial \theta/\partial Y)_{Y=0} \) are known from the solutions of the coupled equations (14) and (15) or equations (22) and (24), respectively, numerical values of the shearing stress in terms of skin-friction coefficient \( C_f Gr^{1/4}/(1 + \gamma) \) from equations (19) or (26) and the heat transfer rate in terms of the Nusselt number \( Nu \) from equations (20) or (27) are calculated from lower stagnation point to upper stagnation point of the circular cylinder. Numerical values of \( C_f Gr^{1/4}/(1 + \gamma) \) and \( NuGr^{-1/4} \) are shown in Tables I-III and Figures 2 and 3. It should be noted that for constant viscosity we recover the
problem that discussed by Merkin (1976) and Nazar et al. (2002) considering $Pr = 1.0$ which is shown in Table I. For the simplicity, the symbol $C_f Gr^{1/4}$ is written instead of $C_f Gr^{1/4}/(1 + \gamma)$.

Figures 2(a) and (b) show the effect of viscous dissipation parameter $Ec^* = (0.0, 0.1, 0.2)$ on the local skin-friction coefficient $C_f Gr^{1/4}$ and the local Nusselt number $NuGr^{-1/4}$ with the viscosity-variation parameter $\gamma = 1.0$ and $M = 0.5$ while $Pr = 0.73$. From Figure 2(a), it is seen that the effect of viscous dissipation on the local skin-friction coefficient $C_f Gr^{1/4}$ is not significant, but it has influence on the local Nusselt number $NuGr^{-1/4}$, so the viscous dissipation term should not be exclude in the heat transfer prediction.

| Table I. |
| Comparison of the present numerical values of $C_f Gr^{1/4}/(1 + \gamma)$ and $NuGr^{-1/4}$ with those of Merkin (1976) and Nazar et al. (2002) while $Pr = 1.0$, $Ec^* = 0.0$ and $\gamma = 0.0$ |
|-----|---------------|----------------|-------------------|------------------|---------------|----------------|-------------------|------------------|
| 0.0 | 0.00000       | 0.00000        | 0.00000           | 0.00000          | 0.4214        | 0.4214        | 0.42161           | 0.42143          |
| $\pi/6$ | 0.37609       | 0.37685        | 0.23947           | 0.23999          | 0.18322       | 0.18366       | 0.11516           | 0.11545          |
| $\pi/3$ | 0.67697       | 0.67927        | 0.43106           | 0.43260          | 0.32984       | 0.33108       | 0.20737           | 0.20819          |
| $2\pi/3$ | 0.95791       | 0.9545         | 0.95726           | 0.95726          | 0.3745        | 0.3741       | 0.1916            | 0.19116          |
| $\pi$ | 0.3391        | 0.3265         | 0.33285           | 0.33590          | 0.1945        | 0.1916        | 0.19344           |                  |

<p>| Table II. |
| The results of $C_f Gr^{1/4}$ for different values of the viscosity-variation parameter $\gamma$ while $M = 0.5$, $Ec^* = 0.1$ and $Pr = 0.73$ |</p>
<table>
<thead>
<tr>
<th>$X$</th>
<th>$\gamma = 0.0$</th>
<th>$\gamma = 1.0$</th>
<th>$\gamma = 3.0$</th>
<th>$\gamma = 5.0$</th>
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</thead>
<tbody>
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<td>DNS</td>
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</tbody>
</table>

<p>| Table III. |
| The results of $NuGr^{-1/4}$ for different values of the viscosity-variation parameter $\gamma$ while $M = 0.5$, $Ec^* = 0.1$ and $Pr = 0.73$ |</p>
<table>
<thead>
<tr>
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</tbody>
</table>
The effect of the magnetic parameter \( M = (0.0, 0.2, 0.5, 0.8, 1.0) \) on the local skin-friction coefficient \( C_f Gr^{1/4} \) and the local Nusselt number \( NuGr^{-1/4} \) with the viscosity-variation parameter \( \gamma = 1.0 \) and for \( Pr = 0.73 \) (air at 20°C and 1 atm pressure) are illustrated in Table II. Here it is noticed that the agreement between the results obtained by using the Keller box method (IFDM) and the DNS is excellent. It can easily be seen that with the effect of magnetic parameter \( M \) leads to decrease the local skin-friction coefficient \( C_f Gr^{1/4} \) and the local Nusselt number \( NuGr^{-1/4} \). This phenomenon can be understood from the fact that the increasing values of magnetic parameter \( M \), the Lorentz force, which opposes the flow, that means, decrease the velocity and temperature gradient and hence the local skin-friction coefficient \( C_f Gr^{1/4} \) and the local Nusselt number \( NuGr^{-1/4} \) decrease. The increasing values of \( M \) increases the fluid temperature within the boundary layer and the associate thermal boundary layer becomes thicker.

The numerical values of the skin-friction coefficient, \( C_f Gr^{1/4} \) and the local Nusselt number \( NuGr^{-1/4} \), against the curvature parameter \( x \) for different values of viscosity-variation parameter \( \gamma = (0.0, 0.5, 1.0, 2.0, 5.0) \) while \( Pr = 0.73 \) (air at 20°C and 1 atm pressure) are entered in Tables II and III, respectively. It is further notice that, the

**Figure 2.**
(a) Skin-friction coefficient; (b) rate of heat transfer for different values of \( Ec^* \) while \( Pr = 0.73, M = 0.5 \) and \( \gamma = 1.0 \)

**Figure 3.**
(a) Skin-friction coefficient; (b) rate of heat transfer for different values of while \( Pr = 0.73, Ec^* = 0.1 \) and \( \gamma = 1.0 \)
agreement between the results obtained by the IFDM and the DNS is excellent. With the increasing values of the viscosity-variation parameter, it is seen that the values of skin-friction coefficient $C_fGr^{1/4}$ and the Nusselt number $NuGr^{1/4}$ decrease.

For increasing values of $\gamma$, the viscosity of the fluid within the boundary layer increases which retards the fluid motion, as a result the corresponding skin-friction coefficient $C_fGr^{1/4}$ decreases. For increasing values of the viscosity-variation parameter, the temperature of the fluid increases which is shown in Figure 5(b). Since the temperature of the fluid increases, the corresponding temperature difference between the surface and the fluid enhances. Due to higher temperature of the fluid the rate of heat transfer that means the Nusselt number $NuGr^{1/4}$ decreases. For example, at $x = \pi/2$, the skin-friction coefficient $C_fGr^{1/4}$ and the local Nusselt number $NuGr^{1/4}$ decrease by 59.55 and 16.79 percent, respectively, as $\gamma$ increases from 0.0 to 5.0.

Attention is now given to the effects of pertinent parameters on the dimensionless velocity and temperature in the flow field, computed only by the implicit finite difference (IFDM) method, and these are presented graphically in Figures 4 and 5.

Figure 4(a) and (b) shows results for the velocity and temperature profiles, based on equations (14) and (15) with the boundary conditions (16), for different values of magnetic parameter $M = (0.0, 0.2, 0.5, 0.8, 1.0)$ plotted against $y$ at $x = \pi/3$ having Prandtl number $Pr = 0.73$ with $\gamma = 1.0$. From Figure 4 it is seen that, as the magnetic parameter $M$ increases, the velocity profile decreases and the temperature profile increases slightly. The reason of this practical scenario is that the interaction of the magnetic field and the moving electric charge carried by the fluid induces a force which tends to oppose the fluid motion. But near the surface of the cylinder, velocity increases and then decreases slowly and finally approaches to zero according to outer boundary condition. This implies that there exists a local maximum of the velocity within the boundary layer.

Figure 5(a) and (b) shows the velocity and temperature distribution against the variable $y$ for different values of the viscosity-variation parameter $\gamma = (0.0, 1.0, 2.0, 3.0, 5.0)$ at $x = \pi/3$ while $Pr = 0.73$ and $M = 0.5$. It can be observed that the velocity decreases and temperature distribution increases with the increasing values of the

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**Figure 4.**
(a) Velocity and (b) temperature distribution for different values of $M$ while $Pr = 0.73$, $Ec^{*} = 0.1$ and $\gamma = 1.0$ at $x = \pi/3$
viscosity-variation parameter, $\gamma$. It should be noted that at each value of the viscosity-variation-parameter $\gamma$, the velocity profile has a local maximum value within the boundary layer. The maximum values of the velocity are 0.33724, 0.29521, 0.26986, 0.25221, 0.22764 at $y = 1.23788, 1.65930, 1.99188, 2.17676, 2.48059$ for $\gamma = 0.0, 1.0, 2.0, 3.0, 5.0$, respectively. The maximum velocity decreases by 32.66 percent as $\gamma$ increases from 0.0 to 5.0. It also be concluded that the velocity boundary layer and the thermal boundary layer thicknesses enhance for large values of $\gamma$.

5. Conclusions
The effect of temperature dependent viscosity on the MHD natural convection boundary layer flow from an isothermal horizontal circular cylinder has been investigated. Numerical solutions of the equations governing the flow are obtained by using the very efficient IFDM together with Keller box scheme and by a DNS. From the present investigation the following conclusions may be drawn:

- Increasing the values of the magnetic parameter $M$ and viscosity-variation parameter $\gamma$ leads to decrease the local skin-friction coefficient $C_{fGr}^{1/4}$ and the local Nusselt number $NuGr^{-1/4}$.

- It is seen the velocity distribution decreases as well as the temperature distribution increases with the increasing values of the magnetic parameter $M$ and viscosity-variation parameter.

- The results have demonstrated that the assumption of constant fluid properties may introduce severe errors in the prediction of the surface shearing stress and the rate of heat transfer.

References


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